M.Sc. (Mathematics) 3rd Semester

DISCRETE MATHEMATICS—I

Paper—MATH-575

Time Allowed—Three Hours]   [Maximum Marks—100

Note :— The question paper will consist of **FIVE** units. Each unit will contain **FOUR** questions. The candidates are required to attempt **TWO** questions from each unit. Each question carries equal marks.

UNIT—I

1. (a) Define Hasse diagram with example.

    (b) Explain partial ordering relation with example.

2. Show that the relation '□' in the set of 2×2 invertible matrices with real entries given by A □ B iff $B = A^{-1}$ is symmetric but not reflexive. Is it transitive ?

3. (a) How many friends must you have to guarantee that at least five of them will have birthdays in the same month ?

    (b) State and prove Generalized pigeonhole principle. Give an example of it.

4. (a) State and prove principle of inclusion and exclusion for two sets.

(b) Let A and B be two finite sets such that :

$n(A - B) = 25$, $n(A \cup B) = 100$, $n(A \cap B) = 40$. Find $n(B)$.

UNIT—II

5. (a) What is difference between conjunctive and disjunctive statements ?

(b) Give an example of Conditional and Biconditional operators.

6. Test the validity of the argument $(S_1, S_2; S)$ where $S_1 : p \wedge q$, $S_2 : \Box\ p$, and $S : q$.

7. By preparing truth table; verify that :

(i) $p \wedge q \rightarrow (p \vee q)$ is a tautology,

(ii) $p \wedge q \wedge \Box\ (p \vee q)$ is a contradiction.

8. (a) Express the following statements in symbols :

(i) Sham is honest and hard working.

(ii) Either Sham is honest or he will pass.

(iii) Sham is honest but will not pass.

(b) Define Truth table with examples.

UNIT—III

9. If f is a homomorphism from a semi group (G, *) to a semi group (G', o). If G_1 is a sub semi group of (G, *) then image of G_1 under give function is a sub semi group of (G', o).

10. Define monoids with example. Let (G, *) and (G', o) be monoids with identities e and i respectively. Let f : G → G' be a homomorphism from (G, *) onto (G', o) then f(e) = i.

11. State and prove fundamental theorem of semi group homomorphism.

12. (a) Show that the intersection of two congruence relations on a semi group is a congruence relation.

    (b) Prove that intersection of two sub monoids of a monoid is a sub monoid of a monoid.

UNIT—IV

13. Explain Phase structure grammars with examples.

14. Explain the concept of derivation sentential forms with examples.

15. Explain the concept of language generated by grammar with suitable examples.

16. Explain context free grammar and languages with examples.

UNIT—V

17. Solve the recurrence relation $S_n - 4S_{n-1} + 4S_{n-2} = 2^n$.

18. Explain the method to find the generating function and sequence from its recurrence relation. Also solve write sequence whose generating function is

$$\frac{6 - 29z}{1 - 11z + 30z^2}.$$

19. Solve :

$$S_n - 6S_{n-1} + 5S_{n-2} = 0 \text{ if } S_0 = 2, \ S_1 = 2.$$

20. (a) Define Recursion, Recurrence and Telescopic form of sequence with example.

    (b) State and prove Recursion theorem.